Fiscal Policy in Small Open Economies: The International Intertemporal Keynesian Cross*

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Abstract

A recent literature studies fiscal policy with realistic intertemporal marginal propensities to consume and the absence of Ricardian equivalence. They show that fiscal policy boosts consumption and is effective when monetary policy is passive: (i) Cumulative fiscal multipliers are above one and (ii) fiscal policy is fully self-financing. I show that these results do not extend to small open economies (SOEs). In SOEs, the initial debt-fueled rise in consumption is offset by a subsequent drop to pay back foreign debt, so (i) the cumulative fiscal multiplier is exactly one, and (ii) fiscal deficits are not fully self-financing.

Keywords: Fiscal Policy, international business cycles, heterogeneous households
JEL Codes: E32, E62, F41

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1 Introduction

A recent literature studies fiscal policy in the absence of Ricardian equivalence. They find that with empirically realistic intertemporal marginal propensities to consume (iMPCs), increased government spending causes consumption to increase, boosting output and fiscal multipliers. Two central results for closed economies with a fixed real interest rate are:

1. The cumulative fiscal multiplier is greater than one (Auclert et al. 2023).

2. Fiscal deficits are fully self-financing (Angeletos et al. 2024).

I re-visit these results in a small open economy (SOE) using an International Intertemporal Keynesian Cross (IIKC), which nests the closed economy of Auclert et al. (2023) when the degree of openness is zero. I find starkly different results when the degree of openness is strictly positive:

1. The cumulative fiscal multiplier is exactly one.

2. Fiscal deficits are less than fully self-financing.

Why is fiscal policy less effective in an SOE? As in the closed economy, expansionary fiscal policy initially generates a rise in consumption. Thus, there is initial crowding-in and the impact fiscal multiplier is above one. The rise in consumption is financed by borrowing from abroad. This current account deficit has to be repaid, which households do by cutting consumption. I show analytically that the drop in consumption exactly offsets the initial rise, such that the consumption is unchanged in cumulative present value terms. This implies a unit cumulative fiscal multiplier.

The zero response of consumption in present value terms and the unit cumulative fiscal multiplier are robust: They hold for (i) any path of government spending, (ii) any timing of financing, (iii) any degree of openness, and (iv) any household behavior satisfying a standard budget constraint and transversality condition.

This has implications for our understanding of the determinants of fiscal multipliers. For instance, the path of primary deficits plays no role in determining the cumulative fiscal multiplier in the SOE, while they play a central role in the closed economy. Additionally, the cumulative fiscal multiplier is the same for a heterogeneous agent model (incomplete markets) and a representative agent model (complete markets) in an SOE, while they are drastically different in a closed economy.

Lastly, I explore some extensions where the cumulative multiplier is not always one. This is possible with (i) less than full financing, (ii) multiple countries that are not collectively small, and (iii) active monetary policy. I provide analytical formulae for the cumulative fiscal multiplier in these cases.

Related Literature Firstly, my paper contributes to the literature on fiscal policy in HANK models. Apart from Auclert et al. (2023) and Angeletos et al. (2024), analytical results for closed economy have in particular been provided by Challe and Ragot (2010), Acharya and Dogra (2020), Bilbiie (2021), and Broer et al. (2021). I differ by focusing on an SOE. Hagedorn et al. (2019) provides a quantitative analysis of fiscal policy in HANK models for a closed economy. No similar analysis has been done for an SOE. My analytical results provide the basis for such an exploration.

Secondly, my paper contributes to the growing literature on open economy HANK models, see e.g. Ferra et al. (2020), Auclert et al. (2021c), Guo et al. (2023), Oskolkov (2023) and Druedahl et al. (2022). My analytical results for fiscal policy are unique.

Structure I split my paper into two parts. In Section 2, I derive analytical results on cumulative fiscal multipliers in an SOE using an International Intertemporal Keynesian Cross. In section 3, I derive analytically the degree of self-financing of fiscal deficits in this economy. Finally, I conclude in Section 4.

2 Fiscal Multipliers in an SOE

2.1 The International Intertemporal Keynesian Cross

In this section, I start with the Intertemporal Keynesian Cross (IKC) with passive monetary policy in the sense of a fixed real interest rate from Auclert et al. (2023). Instead of considering a closed economy, I consider a SOE. This yields the *International* Intertemporal Keynesian Cross (IIKC), which I use to study fiscal multipliers.[1]

Time is discrete, $t = 0, 1, \dots$, and I consider a first-order approximation around a steady state, denoted $dX_t \approx X_t - X_{ss}$ for any variable X. I represent the model in the sequence space following the seminal contribution of Auclert et al. (2021b) by stacking these deviations into a vector, $d\boldsymbol{X} = (dX_0, dX_1, \dots)'$.

1. Auclert et al. (2021c) study a similar IIKC.

Output in the SOE is Y_t and goes to domestic consumers, $C_{H,t}$, foreign consumers, $C^*_{H,t}$, and the government, G_t:

$$d\boldsymbol{Y} = d\boldsymbol{C_H} + d\boldsymbol{C^*_H} + d\boldsymbol{G}. \tag{1}$$

Domestic consumption of home goods is given by

$$d\boldsymbol{C_H} = (1-\alpha)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}), \tag{2}$$

where α is the degree of openness, $\boldsymbol{M}$ is the matrix of intertemporal marginal propensities to consume (iMPCs), and T_t is taxes. This is the natural extension of Auclert et al. (2023) to an SOE, nesting their closed economy consumption function for $\alpha = 0$. Its micro-foundation is standard and is shown in detail in Appendix A.1. Here the baseline calibration used when presenting numerical results is also explained.

I let foreign consumption of domestic goods be fixed, $d\boldsymbol{C^*_H} = \mathbf{0}$. This can be micro-founded by a small domestic economy and a fixed real interest rate, which together with UIP implies a fixed real exchange rate, as shown in Appendix A.1. Inserting this and eq. (2) into eq. (1) yields the International IKC (IIKC):

$$d\boldsymbol{Y} = (1-\alpha)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + d\boldsymbol{G}. \tag{3}$$

Lastly, I use two assumptions of a more technical character: A standard transversality condition and a standard aggregate household budget constraint:

$$\lim_{t\to\infty} \frac{A_t}{(1+r)^t} = 0, \tag{4}$$

$$C_t + A_t = (1+r)A_{t-1} + Z_t, \tag{5}$$

where $Z_t \equiv Y_t - T_t$ is real disposable income, A_t is end-of-period assets, C_t is consumption, and r is the real interest rate. Both hold in all standard models, including a representative agent model and heterogeneous agent models.

I now state my first result regarding the solution of the IIKC.

Proposition 1 (Existence and uniqueness of a solution)**.**

- *In a closed economy ($\alpha = 0$), there is not a unique solution for $d\boldsymbol{Y}$.*
- *In an SOE ($0 < \alpha < 1$), there is a unique solution for $d\boldsymbol{Y}$.*

Proof. The proof for the first bullet is given in Auclert et al. (2023). For the second bullet, it follows from $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]\boldsymbol{x} = \boldsymbol{0}$ only having the solution $\boldsymbol{x} = 0$. See Appendix B.1 for details. □

The lack of a unique solution in the closed economy is also given in Auclert et al. (2023). The fact that a unique solution exists in the SOE is an attractive property, as one avoids having to pick a particular solution.[2] In addition to the solution being unique in an SOE, it is also straightforward to obtain it. In particular, the unique solution is simply given by solving the linear system of equations in eq. (3) using standard methods, yielding:

$$d\boldsymbol{Y} = [I - (1-\alpha)\boldsymbol{M}]^{-1}\left[d\boldsymbol{G} - (1-\alpha)\boldsymbol{M}d\boldsymbol{T}\right].$$

This is possible in the SOE because $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]^{-1}$ exists, while it is not possible in the closed economy because $[\boldsymbol{I} - \boldsymbol{M}]^{-1}$ does *not* exist. This is exactly what guarantees uniqueness in the SOE, but not in the closed economy. Intuitively, the uniqueness in the SOE stems from the fact that a fraction α of consumption goes abroad, which rules out explosive equilibria. When comparing to the closed economy, I focus on the unique solution that converges to steady state.

2.2 Fiscal Multipliers

With the IIKC, I now turn my attention to cumulative fiscal multipliers. Following Auclert et al. (2023), Ramey (2016), Mountford and Uhlig (2009), and others, I compute the cumulative fiscal multiplier following a government spending shock as the cumulative present value change in output per unit of cumulative present value change in government spending:

$$\mathcal{M} \equiv \frac{\sum_{t=0}^{\infty} \frac{dY_t}{(1+r)^t}}{\sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t}} = \frac{\boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}' d\boldsymbol{G}}, \tag{6}$$

with $\boldsymbol{q}' \equiv (1, (1+r)^{-1}, (1+r)^{-2}, \dots)$, where r is the real interest rate.

2. The IKC in the closed economy, however, does have a unique solution that satisfies $\lim_{t\to\infty} dY_t = 0$. Picking this particular solution can be justified as this is the solution one obtains when considering a Taylor rule with the coefficient of inflation approaching one from above, c.f. Auclert et al. (2023).

The government's budget constraint is

$$B_t = (1+r)B_{t-1} + \mathrm{PD}_t, \tag{7}$$

where B_t is the government's real debt and $\mathrm{PD}_t = G_t - T_t$ is the primary deficit. As in Auclert et al. (2023), I restrict my attention to the case of $\lim_{t\to\infty} B_t/(1+r)^t = 0$. With this setup, I state my second result.

Proposition 2 (Cumulative fiscal multipliers)**.**

- *In a closed economy, the cumulative fiscal multiplier is not* 1 *in general. In particular, it is greater than* 1*,* $\mathcal{M} > 1$*, in models with realistic iMPCs like standard heterogeneous agent models.*
- *In an SOE, the cumulative fiscal multiplier is exactly* 1*:* $\mathcal{M} = 1$*.*

Proof. The closed economy result in the first bullet is from Auclert et al. (2023). The second bullet follows from pre-multiplying the IIKC in eq. (3) by $\boldsymbol{q}'$ while using $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}$ and $\boldsymbol{q}'d\boldsymbol{G} = \boldsymbol{q}'d\boldsymbol{T}$. See Appendix B.2 for details. □

Proposition 2 is a striking result. It implies that there is an analytical formula for the cumulative fiscal multiplier, which is rarely the case in a closed economy. Additionally, this multiplier is exactly one and independent of household behavior, which is central in the closed economy. Furthermore, Proposition 2 implies a discontinuity shown in Figure 1: For any degree of openness $\alpha > 0$, the cumulative fiscal multiplier is 1, even as α becomes closer and closer to 0. However, when α is *exactly* 0, the cumulative fiscal multiplier jumps discontinuously to > 1, as shown in Figure 1.

So why can large MPCs not sustain a boom following a government spending shock in an SOE, while they do in a closed economy? Note that consumption *can* initially rise in both the closed economy and the SOE, as shown in Figure 2, which shows impulse response functions following a government spending shock in the model from Appendix A.1. In the closed economy, consumption then returns to steady state, while consumption in the SOE drops below steady state. This is because government spending implies running a current account deficit in the SOE, building a negative net foreign asset position. Eventually, this has to be repaid by cutting back consumption. This drop is large enough to exactly offset the initial rise, such that cumulative consumption is exactly unchanged in present value terms, as shown in Corollary 1, and the cumulative fiscal multiplier is exactly 1.

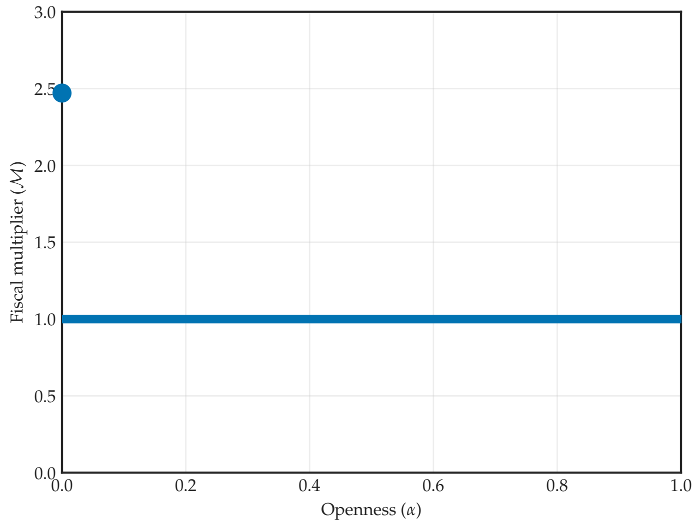

Figure 1: The cumulative fiscal multiplier for different degrees of openness

Note: The figure shows the cumulative fiscal multiplier for different values of α. The cumulative fiscal multiplier in the closed economy is from the model in Appendix A.1 with $\alpha = 0$.

There is no current account deficit in the closed economy, as there are no foreigners to hold the government's debt. Instead, domestic households own the government debt. When the debt is paid back to them, the households eventually spend at home, sustaining the boom.

Corollary 1. *The present value consumption change in an SOE following a $d\boldsymbol{G}$ shock is*

$$\sum_{t=0}^{\infty} \frac{dC_t}{(1+r)^t} = 0.$$

Proof. Pre-multiplying $d\boldsymbol{C} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T})$ by $\boldsymbol{q}'$ gives $\boldsymbol{q}'d\boldsymbol{C} = \boldsymbol{q}'d\boldsymbol{Y} - \boldsymbol{q}'d\boldsymbol{T} = 0$, using $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$, $\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G}$, and $\mathcal{M} = 1$. □

While it makes sense that the cumulative fiscal multiplier is smaller in the SOE than in the closed economy, it is perhaps still surprising that it is *exactly* 1. This is the case because the cumulative present-value MPC is 1:

$$\frac{\partial(q'd\boldsymbol{C})}{\partial(q'd\boldsymbol{Z})} = 1.$$

Intuitively, this just means that any increase in household income has to be spent at some point in time in a present value sense. This is not an assumption or implication of the particular household behavior. In fact, it only requires a standard household budget constraint and transversality condition. This makes Proposition 2

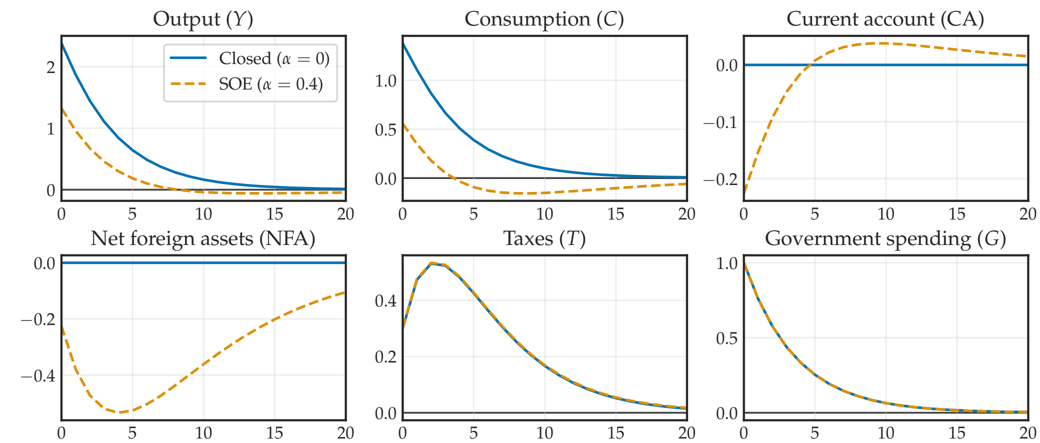

Figure 2: IRFs to a government spending shock in a closed economy and an SOE

Note: The figure shows impulse response functions (IRFs), i.e. $100 \cdot d\boldsymbol{X}$, for various $\boldsymbol{X}$ from the model in Appendix A.1. The x-axis shows years after the shock.

general: It holds for a broad class of assumptions on household behavior, including a representative agent and standard heterogeneous agent specifications.

Note that with a representative agent, consumption is unchanged at all points in time, $dC_t = 0$, implying $dY_t = dG_t$ (c.f. Appendix A.4). Here, the path of dT_t does not matter for dC_t at all. This is the standard result of Ricardian equivalence.

2.3 Breaking the Unit Multiplier

Having established a unit cumulative fiscal multiplier in the baseline setup, I now consider possible model changes that can break the unit fiscal multiplier.

2.3.1 Partial Financing

So far, the fiscal policy shock has been fully financed in an intertemporal sense: $\boldsymbol{q}'d\boldsymbol{G} = \boldsymbol{q}'d\boldsymbol{T}$. This is a requirement in the closed economy due to the transversality condition and asset market clearing, but not in the SOE as $A_t \neq B_t$ is possible.[3] I now consider *partially* financed shocks. In particular, I replace $\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G}$ by:

$$\boldsymbol{q}'d\boldsymbol{T} = f\boldsymbol{q}'d\boldsymbol{G}, \tag{8}$$

3. Note first the transversality condition, $(1+r)^{-t}A_t \to 0$ as $t \to \infty$. Using asset market clearing in the closed economy ($A_t = B_t$) then gives $(1+r)^{-t}B_t \to 0$, which requires $\boldsymbol{q}'d\boldsymbol{G} = \boldsymbol{q}'d\boldsymbol{T}$. In the SOE, A_t can be different from B_t, so $(1+r)^{-t}A_t \to 0$ does not imply $(1+r)^{-t}B_t \to 0$, so $\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G}$ is not required. Intuitively, the SOE opens a negative NFA position if it does not finance fiscal deficits.

where $f \in [0, 1]$ is the degree of financing. $f = 1$ represents full financing, $f = 0$ no financing, and $f \in (0, 1)$ somewhere in-between. In this case, $\lim_{t\to\infty} B_t/(1 + r)^t > 0$.

Proposition 3. *The cumulative fiscal multiplier of a fiscal policy shock financed by degree $f \in [0, 1]$ is*

$$\mathcal{M} = 1 + \frac{(1-\alpha)(1-f)}{\alpha}.$$

Proof. The proof follows the proof of Proposition 2, except using eq. (8) instead of $\boldsymbol{q}' d\boldsymbol{G} = \boldsymbol{q}' d\boldsymbol{T}$. See Appendix B.3 for details. □

This nests the case of fully financed shocks for $f = 1$, implying $\mathcal{M} = 1$. For $f < 1$, the multiplier is greater than 1, and the multiplier is decreasing in the degree of openness. Note that the path of dT_t does not matter for a given net present value.

2.3.2 Two SOEs

So far, I have considered a single SOE. I now consider two SOEs that affect each other but are collectively small compared to the rest of the world. They buy shares $\alpha \in (0, 1)$ and $\alpha^* \in (0, 1)$ from the other country, and shares $\epsilon \geq 0$ and $\epsilon^* \geq 0$ from the rest of the world. The sequence space consumption functions are

$$\begin{aligned} d\boldsymbol{C_H} &= (1-\alpha-\epsilon)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}), & d\boldsymbol{C_F} &= \alpha\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}), \\ d\boldsymbol{C_H^*} &= \alpha^*\boldsymbol{M^*} d\boldsymbol{Y}^*, & d\boldsymbol{C_F^*} &= (1-\alpha^*-\epsilon^*)\boldsymbol{M^*} d\boldsymbol{Y}^*, \end{aligned}$$

where $\boldsymbol{M^*}$ is the iMPC matrix in the second country. C_F and C_F^* denote the consumption of foreign goods at home and abroad, respectively. With Y_t^* denoting output abroad, goods market clearing for both countries is given by

$$d\boldsymbol{Y} = d\boldsymbol{C_H} + d\boldsymbol{C_H^*} + d\boldsymbol{G}, \quad d\boldsymbol{Y}^* = d\boldsymbol{C_F} + d\boldsymbol{C_F^*}.$$

Stacking this on vector form gives the multi-country IIKC:

$$\begin{pmatrix} d\boldsymbol{Y} \\ d\boldsymbol{Y^*} \end{pmatrix} = \begin{pmatrix} (1-\alpha-\epsilon)\boldsymbol{M} & \alpha^*\boldsymbol{M^*} \\ \alpha\boldsymbol{M} & (1-\alpha^*-\epsilon^*)\boldsymbol{M^*} \end{pmatrix} \begin{pmatrix} d\boldsymbol{Y} - d\boldsymbol{T} \\ d\boldsymbol{Y^*} \end{pmatrix} + \begin{pmatrix} d\boldsymbol{G} \\ 0 \end{pmatrix}. \tag{9}$$

I now consider the cumulative fiscal multiplier in this setup.

Proposition 4. *For all possible $\alpha \in (0, 1)$ and $\alpha^* \in (0, 1)$, the cumulative fiscal multiplier is 1, $\mathcal{M} = 1$, when $\epsilon > 0$ or $\epsilon^* > 0$ (or both).*

Proof. The proof is like the proof of Proposition 2, except starting with the multi-country IIKC in eq. (9) and pre-multipliying by $(\boldsymbol{q}', \boldsymbol{q}')$. See Appendix B.4 for details. □

This shows that the cumulative fiscal multiplier remains at 1 as long as either of the two countries buys at least some small $\epsilon > 0$ or $\epsilon^* > 0$ from the rest of the world. In this case, the intuition from Proposition 2 still holds: The domestic boom is financed by borrowing from abroad, which is repaid by cutting consumption. When, $\epsilon = 0$, the borrowing is only from domestic households (as in the closed economy) and households in the other SOE. Both types of households buy from the domestic economy, which sustains the boom as in the closed economy.[4]

2.3.3 Active Monetary Policy

So far, the nominal interest rate has responded one-to-one with inflation such that the real interest rate is fixed. This is a natural starting point as it strikes "a middle ground between loose policy (like at the zero lower bound) and tight policy (like with an active Taylor rule)", as argued in Auclert et al. (2023). I now relax this assumption and consider active monetary policy according to the following monetary policy rule:

$$i_t = i_{ss} + \phi_\pi \pi_{t+1}, \tag{10}$$

where i_t is the nominal interest rate and π_t is inflation — see the full model in Appendix A.1. In this case, the cumulative fiscal multiplier is as follows.

Proposition 5. *The cumulative fiscal multiplier is*

$$\mathcal{M} = 1 + \underbrace{\frac{1-\alpha}{\alpha}\frac{A_{ss}}{1+r}\frac{\boldsymbol{q}'d\boldsymbol{r}}{\boldsymbol{q}'d\boldsymbol{G}}}_{\text{Wealth effect}} + \underbrace{\left[\frac{\chi}{1-\alpha} - 1\right]\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}}}_{\text{Exchange rate}} + \underbrace{G_{ss}\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}} - \frac{1-\alpha}{\alpha}PD_{ss}\sum_{t=0}^{\infty} dq_t}_{\text{Financing cost}},$$

where χ is the trade elasticity defined in Appendix B.5, Q_t is the real exchange rate, and dq_t is a perturbation of $q_t = (1+r_0)^{-1} \dots (1+r_{t-1})^{-1}$ around $(1+r)^{-t}$.

Proof. The proof follows the proof of Proposition 2, using a generalized IIKC with

4. I only prove only that $\mathcal{M} = 1$ when $\alpha \in (0,1)$, $\alpha^* \in (0,1)$, and $\epsilon > 0$ or $\epsilon^* > 0$. I do *not* prove anything about $\mathcal{M}$ when $\epsilon = \epsilon^* = 0$. I have numerically verified that $\epsilon = \epsilon^* = 0$ gives $\mathcal{M} \neq 1$ in general.

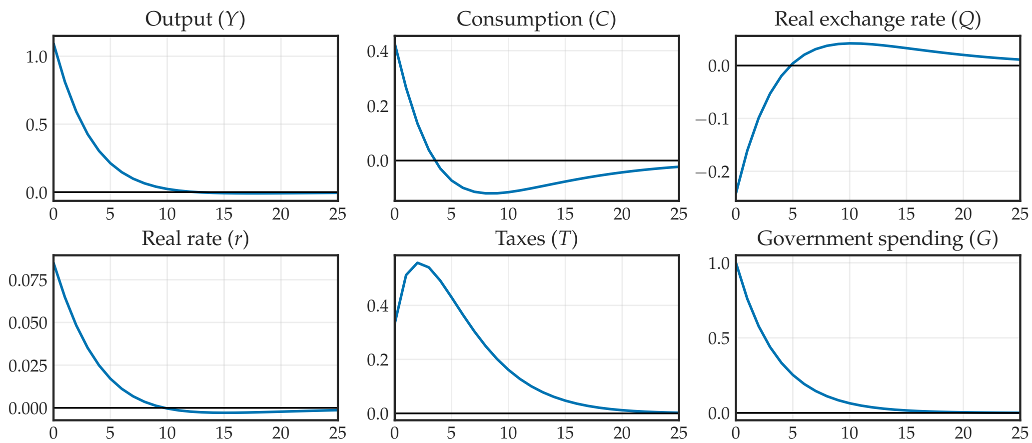

Figure 3: IRFs to a government spending shock with active monetary policy

Note: The figure shows $100 \cdot d\boldsymbol{X}$ for various $\boldsymbol{X}$ from the model in Appendix A.1 with $\phi_\pi = 1.5$ and $\kappa = 0.03$. The NFA is $A_t - B_t$, while the current account is the first difference of the NFA. The x-axis shows years after the shock.

the added real interest rate effect and using that $\boldsymbol{q}'(\partial \boldsymbol{C}/\partial \boldsymbol{r}) = A_{ss}(1+r)^{-1}\boldsymbol{q}'$. See Appendix B.5 for details. □

In the case with a constant real interest rate, $d\boldsymbol{r} = d\boldsymbol{Q} = 0$ and $dq_t = 0$, so $\mathcal{M} = 1$. With active monetary policy, the real interest rate appreciates due to higher domestic demand as shown in Figure 3.

The higher real interest rate creates the following two new channels.

1. **Wealth effect.** A higher real interest rate implies that households are wealthier, stimulating consumption and boosting output in general equilibrium.

2. **Exchange rate.** A higher real interest rate appreciates the real exchange rate by capital inflow. This has two effects. First is expenditure switching: An appreciation makes consumers substitute away from domestic goods, lowering output. Second is the real income channel (Auclert et al. 2021c): An appreciation makes domestic consumers richer in real terms, boosting consumption and output. I note that both (i) the sign of the response of the real exchange rate and (ii) which of the two channels dominates depends on the calibration.

3. **Financing cost.** A higher real interest rate and appreciated real exchange rate make it more expensive for the government to finance deficits through two channels. The first is that a higher real interest rate makes it more expensive to pay back debt. The second is that an appreciated real exchange

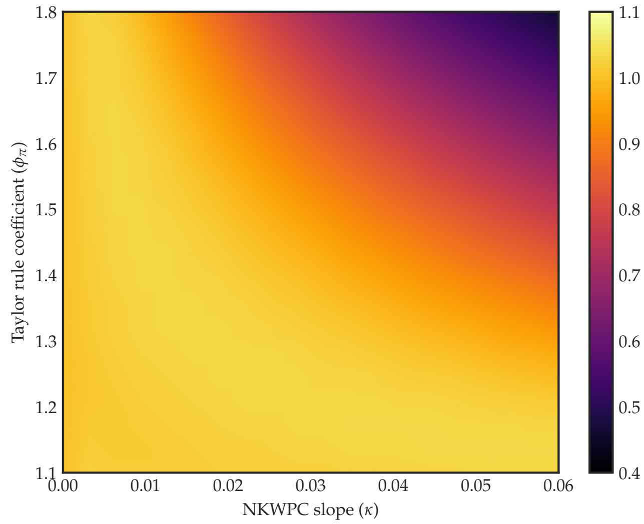

Figure 4: Cumulative fiscal multiplier with active monetary policy

Note: The figure shows $\mathcal{M}$ for different values of ϕ_π and κ in the model from Appendix A.1.

rate means that government spending is more expensive. Both imply that the government has to increase taxes more to finance government spending, lowering consumption and output.

The wealth effect is close to 0 for my calibration because households do not hold many assets ($A_{ss} = 0.21$, following Auclert et al. 2023), and the financing cost is also unimportant. Thus, movements in the real exchange rate are what matter. I note that the empirical literature is not clear on what happens with the real exchange rate following a fiscal policy shock. Some contributions find that the real exchange rate depreciates following positive fiscal policy (Monacelli and Perotti 2010, Ravn et al. 2012, and Kim 2015), while others find an appreciation (Miyamoto et al. 2019 and Born et al. 2023). In Figure 4, I show that the cumulative fiscal multiplier remains close to 1 for a broad range of calibrations.

In Appendix A.3, I show that the results are similar with a fixed exchange rate.

3 Self-Financing Fiscal Deficits

3.1 The Definition of Self-Financing

Given that cumulative fiscal multipliers are smaller in an SOE than in a closed economy, a natural question is what the implications are for the degree of self-financing of fiscal deficits. To study this, I repeat a similar analysis as Angeletos et al. (2024) in an SOE. In particular, I extend the SOE in Section 2 with labor taxes, so the government's primary deficit is

$$\mathrm{PD}_t = G_t - T_t - \tau Y_t,$$

and the household's budget constraint is

$$C_t + A_t = (1+r)A_{t-1} + (1-\tau)Y_t - T_t.$$

Iterating on the government's budget constraint yields

$$\frac{B_t}{(1+r)^t} = (1+r)B_{ss} + \sum_{s=0}^{t} \frac{G_s - T_s}{(1+r)^s} - \tau \sum_{s=0}^{t} \frac{Y_s}{(1+r)^s}. \tag{11}$$

The $G_s - T_s$ term is controlled directly by the government, while the τY_s term reflects self-financing: Deficits create a boom that increases labor income and hence labor taxes, reducing the deficit. Following Angeletos et al. (2024), I define the degree of self-financing in the limit as $t \to \infty$ by

$$\nu \equiv \frac{\tau \sum_{t=0}^{\infty} \frac{dY_t}{(1+r)^t}}{\sum_{t=0}^{\infty} \frac{dG_t - dT_t}{(1+r)^t}} = \frac{\tau \boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T})}.$$

This object measures the receipts from labor taxes relative to the fiscal deficit from increased government spending and/or lump-sum taxes. If the labor taxes raised are enough to exactly cover the deficit in a present-value sense, the degree of self-financing is 1, i.e. fiscal deficits are self-financing. In this case, both the present value of primary deficits and the present value of government bonds as $t \to \infty$ are zero, which was assumed directly in Section 2. I show this in Proposition 6. If instead labor taxes do not cover the deficit, the degree of self-financing is less than 1.

Proposition 6. *From the government's budget constraint alone, $\nu = 1$ implies both*

$$\lim_{t \to \infty} \frac{B_t}{(1+r)^t} = 0,$$
$$\boldsymbol{q}' d\boldsymbol{PD} = 0.$$

Proof. This follows from iterating on the government's budget constraint in eq. (11) and using the definition of self-financing. See Appendix B.6 for details. □

3.2 Self-Financing In Closed and Open Economies

With the definition of self-financing in mind, I now provide some results on the financing of deficits in closed and open economies.

Proposition 7. *Consider two fiscal policy shocks: (1) A government spending shock, $d\boldsymbol{G} \neq \boldsymbol{0}$, with fixed lump-sum taxes, $d\boldsymbol{T} = \boldsymbol{0}$, or (2) a lump-sum tax shock, $d\boldsymbol{T} \neq \boldsymbol{0}$, with fixed government spending, $d\boldsymbol{G} = \boldsymbol{0}$. Define their degrees of self-financing as ν_G and ν_T.*

- *In a closed economy, a fiscal deficit is completely self-financing, i.e. $\nu_G = \nu_T = 1$.*
- *In an SOE, $\alpha > 0$, a fiscal deficit is not self-financing, i.e. $\nu_G < 1$ and $\nu_T < 1$:*

$$\nu_G = \frac{\tau}{(1-\alpha)\tau + \alpha}, \qquad \nu_T = \frac{(1-\alpha)\tau}{(1-\alpha)\tau + \alpha}. \tag{12}$$

Proof. The proof follows from extending the IIKC with labor taxes, pre-multiplying by $\boldsymbol{q}'$, and using the definition of self-financing. See Appendix B.7 for details. □

The closed economy result in the first bullet is the main result in Angeletos et al. (2024) when financing is delayed forever. The result implies that the government does not have to reduce transfers or change the tax rate to finance government spending. Instead, the tax receipts from the boom are enough to cover the deficit.

The SOE result in the second bullet implies that this result does not extend to an SOE. In this case, the degree of self-financing is less than perfect. The intuition is similar to Proposition 2: Fiscal policy creates less of a boom when the economy is open. However, in contrast with Proposition 2, the degree of self-financing is not discontinuous at $\alpha = 0$, but converges to $\nu = 1$ as α approaches 0 from above.

The result for the SOE is striking because it contains simple, intuitive analytical formulae for the degrees of self-financing, something that does not exist for the closed economy. To understand these formulae, Figure 5 shows ν_G and ν_T for different values of α and τ. Intuitively, the degree of self-financing is increasing in the tax rate, τ, simply because more taxes are paid. Additionally, the degree of self-financing is decreasing in the degree of openness, α, because more consumption is "lost abroad".

In the closed economy limit of $\alpha = 0$, there is perfect self-financing (as in Angeletos et al. (2024)): $\nu_G = \nu_T = 1$. How far does the SOE deviate from this closed economy benchmark? If, for instance, $\tau = 0.3$ (Angeletos et al. 2024) and $\alpha = 0.4$ (Auclert et al. 2021c), $\nu_G = 0.52$ and $\nu_T = 0.31$, i.e. there is half or less self-financing.

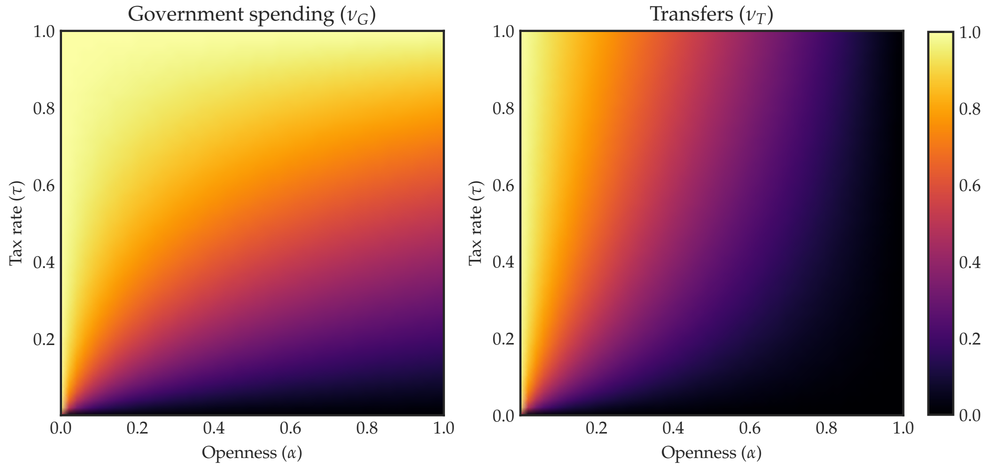

Figure 5: Degrees of self-financing

Note: The left panel shows the degree of self-financing for government spending, i.e. ν_G from eq. (12). The right panel shows the degree of self-financing for transfers, i.e. ν_T from eq. (12). Both panels show the degrees of self-financing for different values of openness (α) on the x-axis and different tax rates (τ) on the y-axis.

Lastly, it is worth considering why ν_G and ν_T differ when the economy is open ($\nu_G \neq \nu_T$), while they are the same in the closed economy ($\nu_G = \nu_T = 1$). In particular, why is $\nu_T = (1-\alpha)\nu_G$, so $\nu_T < \nu_G$ for $\alpha \in (0,1)$? This is because a share $1-\alpha$ of transfers are spent abroad by domestic households, while the government only buys domestic goods. Thus, the degree of self-financing for government spending is larger than for transfers by a factor $(1-\alpha)^{-1}$.

4 Conclusion

I study fiscal policy by re-visiting the closed economy results of Auclert et al. (2023) and Angeletos et al. (2024) in an SOE. I show that fiscal policy is much less effective at stimulating the domestic economy in an SOE than in a closed economy even with realistic intertemporal marginal propensities to consume: Cumulative consumption is unchanged in present value terms, the cumulative fiscal multiplier is exactly 1, and fiscal deficits are not fully self-financing.

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Appendix:

Fiscal Policy in Small Open Economies: The International Intertemporal Keynesian Cross

Jacob Marott Sundram

A Models

A.1 Main Model

In this Appendix, I present a model that micro-founds the results in the paper. I note that many results in the main text are more general than this model. For instance, the consumption function in eq. (2) holds for many different household problems, not just the particular one presented here.

The model presented here can be seen as the model in Auclert et al. (2023) for a small open economy instead of a closed economy, which is essentially the model in Auclert et al. (2021c) with a government. The model in Auclert et al. (2021c) is itself the model in Gali and Monacelli (2005) with heterogeneous households instead of a representative agent.

Households. The household side is characterized by a standard incomplete markets household problem (Aiyagari 1994, Bewley 1979, Huggett 1993, Imrohoroğlu 1989). There is a continuum of households. Each household with assets a, and idiosyncratic earnings e chooses consumption c, and next-period assets a' (where primes denote variables in the next period). They choose this to solve the following dynamic problem:

$$
\begin{aligned}
V_t(a,e) &= \max_{c,a'} u(c) + \beta \mathbb{E}_t \left[V_{t+1}(a',e') \right], \\
&\text{s.t.} \\
c + a' &= (1 + r_{t-1})a + Z_t \frac{e^{1-\theta}}{\mathbb{E}\left[e^{1-\theta}\right]}, \\
a' &\geq 0.
\end{aligned}
$$

The aggregate variables are post-tax labor income, Z_t, and the ex-post real return on assets, r_{t-1}. u and v are the instantaneous utility of consumption and the disutility of labor supply, respectively. $\theta \in [0,1]$ controls tax progressivity following Heathcote et al. (2017). As is standard in the HANK literature (Auclert et al. 2023 and Auclert et al. 2021a), the labor union chooses labor supply, N_t, see Appendix A.1.

Log idiosyncratic earnings, $\log e$, follows an AR(1) process with persistence ρ_e and variance σ_e^2. I discretize this as a Markov chain normalized such that $\mathbb{E}[e] = 1$. Utility of consumption and disutility of labor follow standard functional forms,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \text{and} \quad v(n) = \gamma n^{1+\frac{1}{\phi}},$$

where $\sigma > 0$ is the inverse elasticity of intertemporal substitution, $\phi > 0$ is the Frisch elasticity of labor supply, and $\gamma > 0$ is a normalization constant. Aggregate real disposable income is

$$Z_t = (1-\tau_t) w_t N_t - T_t, \tag{13}$$

where w_t is the real wage rate. The real wage rate is $w_t = W_t / P_t$, where W_t is the nominal wage rate and P_t is the consumer price index (CPI). I denote aggregate consumption and aggregate net assets by C_t and A_t.

Aggregate consumption is split into consumption of home goods, $C_{H,t}$, and foreign goods, $C_{F,t}$:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \tag{14}$$

$$C_{H,t} = \alpha \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t,$$

where $\eta > 0$ is the elasticity of substitution between goods. The CPI is then

$$P_t = \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{15}$$

where $P_{H,t}$ is the price of home goods and $P_{F,t}$ is the price of foreign goods.

Production. I consider the most simple production side possible. Production is linear in labor:

$$Y_t = N_t,$$

and there is perfect competition such that the price equals the marginal cost, which is the wage:

$$P_{H,t} = W_t.$$

When foreign goods are sold abroad, their price in foreign currency is simply set using the nominal exchange rate, E_t:

$$P^*_{H,t} = \frac{P_{H,t}}{E_t}.$$

Public Sector. The government's real budget constraint is

$$B_t = (1 + r_{t-1})B_{t-1} + \mathrm{PD}_t, \tag{16}$$

where

$$\mathrm{PD}_t = \frac{P_{H,t}}{P_t}G_t - T_t - \tau_t w_t N_t$$

is the primary deficit. The government sets a constant labor tax rate, $\tau_t = \tau$, and uses lump-sum transfers to balance the budget according to the following rule from Auclert et al. (2023):

$$dB_t = \phi_B(dB_{t-1} + dG_t).$$

The central bank either sets the nominal interest rate, i_t, such that the real interest rate is constant,

$$r_t = r_{ss},$$

or follows a monetary policy rule:

$$i_t = r_{ss} + \phi_\pi \pi_{t+1}.$$

The nominal and real interest rate is connected through a Fisher equation:

$$1 + i_t = (1 + r_t)(1 + \pi_{t+1}),$$

where $\pi_t = P_t/P_{t-1} - 1$ is CPI inflation.

The Foreign Economy. The home country trades with a foreign economy. Consumption in the foreign economy is fixed at $C_t^* = C_{ss}^*$. The consumption of home goods by foreign consumers is then

$$C_{H,t}^* = \alpha \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\gamma} C_t^*, \tag{17}$$

where $P_t^* = P_{ss}^*$ is the foreign CPI, which is fixed due to the domestic economy being small. The nominal exchange rate is

$$E_t = \frac{P_{F,t}}{P_{F,t}^*}. \tag{18}$$

I define the real exchange rate as

$$Q_t = \frac{E_t P_t^*}{P_t}. \tag{19}$$

The (real) UIP condition arbitraging away returns across economies is given by

$$1 + r_t = (1 + r^*) \frac{Q_{t+1}}{Q_t}, \tag{20}$$

where r^* is the foreign real interest rate.

Labor Union. Labor supply is set by unions subject to quadratic costs of adjusting the nominal wage. This is exactly as in Auclert et al. (2023), who show that the problem yields the following new-Keynesian wage Phillips curve (NKWPC):

$$\pi_t^w (1 + \pi_t^w) = \kappa^w \left(\frac{\gamma N_t^{1/\phi}}{(C_t^v)^{-\sigma} (1 - \theta) Z_t / N_t} - 1 \right) + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w),$$

where γ is chosen such that the NKWPC holds in steady state, $\pi_t^w = W_t / W_{t-1} - 1$ is wage inflation, and C_t^v is a virtual consumption aggregate given by

$$C^v \equiv \left\{ \mathbb{E} \left[\frac{e_{i,t}^{1-\theta}}{\mathbb{E}(e_{i,t}^{1-\theta})} c_{i,t}^{-\sigma} \right] \right\}^{-\frac{1}{\sigma}},$$

where the expectation is taken across the distribution of households. With a fixed real interest rate, this NKWPC does not matter for real outcomes.

Market Clearing. Production of goods Y_t goes to three sources: Domestic consumption, foreign consumption, and (domestic) government spending. Thus, home goods market clearing is given by

$$Y_t = C_{H,t} + C^*_{H,t} + G_t. \tag{21}$$

Assets. Households can hold two assets: Domestic bonds, B_t, and foreign bonds, B^*_t. The NFA is the difference between the value of assets held at home, A_t, and the value of the supply of domestic assets, B_t:

$$\text{NFA}_t = A_t - B_t. \tag{22}$$

In steady state, domestic households hold all domestic bonds, such that the NFA is zero. The current account is the change in the NFA:

$$\text{CA}_t = \text{NFA}_t - \text{NFA}_{t-1}.$$

Steady State. I consider a zero-inflation steady state,

$$\pi_{ss} = \pi_{H,ss} = \pi_{F,ss} = 0,$$

where all prices are one:

$$P_{ss} = P^*_{ss} = P_{H,ss} = P^*_{H,ss} = P_{F,ss} = P^*_{F,ss} = W_{ss} = w_{ss} = E_{ss} = Q_{ss} = 1.$$

I consider a steady state with zero net exports and NFA: $\text{NX}_{ss} = \text{NFA}_{ss} = 0$. I normalize output to 1, $Y_{ss} = N_{ss} = 1$, which implies $Z_{ss} = 1 - T_{ss}$ and $C_{ss} = C^*_{ss} = 1 - G_{ss}$. It follows that $C_{H,ss} = (1-\alpha)C_{ss}$, $C_{F,ss} = \alpha C_{ss}$, $C^*_{H,ss} = \alpha C^*_{ss}$, and $C^*_{F,ss} = (1-\alpha)C^*_{ss}$.

The supply of bonds in steady state is chosen by the government: $B_{ss} = A_{ss}$. The interest rate is calibrated and is common due to arbitrage and zero inflation:

$$i_{ss} = r_{ss} = r^*_{ss} = r.$$

The government chooses government spending, G_{ss}, and labor taxes, τ, in steady state. Lump-sum transfers ensure that bonds are constant in the steady state

$$T_{ss} = r_{ss}B_{ss} + G_{ss} - \tau w_{ss}N_{ss}.$$

Calibration. I simulate the model for some of the figures in the main text. To do this, I use the calibration from Auclert et al. (2023).[5] Two parameters related to openness are not present in their model: α and χ. For α, I follow Auclert et al. (2021c) and set $\alpha = 0.4$. For χ, I choose $\chi = 1$. I note that χ only matters when monetary policy is active.

Parameter	σ	φ	β	ρ_e	σ_e	θ	r
Value	1	1	0.871	0.91	0.92	0.181	0.05
Parameter	B_{ss}	G_{ss}	ϕ_B	α	χ	dG_0	ρ_G
Value	0.21	0.20	0.7	0.4	1.0	0.01	0.76

Table A.1: Calibration

Note: This table shows the calibration of the baseline SOE model.

A.2 Micro-Founding The Consumption Function

In this Appendix, I show how to micro-found the consumption function based on the model in Appendix A.1. Consider the consumption function in the sequence space,

$$C_t = \mathcal{C}_t(\{Z_s\}_{s=0}^{\infty}),$$

where $Z_t = (1 - \tau_t)w_tN_t - T_t$ is post-tax income. In Section 2, I consider only lump-sum taxes: $\tau_t = 0$. As in Auclert et al. (2023), consumption does not depend on the real interest rate as this is fixed (i.e. it is a parameter). Linearizing the consumption function yields

$$d\boldsymbol{C} = \boldsymbol{M}d\boldsymbol{Z}. \tag{23}$$

5. Most parameters come from Table 2 in their paper. The other parameters come from the following. φ and ρ_G come from Table 1. dG_0 comes from pp. 37. G_{ss} comes from pp. 38. Finally, for ϕ_B they consider many different values in Figure 5, so I choose $\phi_B = 0.7$. For larger values, the cumulative fiscal multiplier becomes large in the closed economy.

where the Jacobian

$$\boldsymbol{M} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{Z}}$$

is the iMPC matrix.

A crucial result is that $P_{H,t} = P_t$. To see this, normalize first all prices in steady state to 1. The domestic economy is small, so $P_t^* = P_{F,t}^* = P_{F,ss}^* = P_{ss}^* = 1$, which I use throughout. UIP in eq. (20) with $r_t = r_{ss}$ then gives $Q_{t+1} = Q_t$ so $Q_t = Q_{ss} = 1$. Inserting this into the definition of the real exchange rate in eq. (19) gives $E_t = P_t$. Inserting this into eq. (18) yields $E_t = P_{F,t}$. Combining this with $E_t = P_t$ gives $P_t = P_{F,t}$. Inserting this into the CPI in eq. (15) and solving for $P_{H,t}$ finally yields $P_{H,t} = P_t$. This implies that $Z_t = Y_t - T_t$, which inserted into eq. (23) yields

$$d\boldsymbol{C} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}). \tag{24}$$

Linearizing eq. (14), using $P_{H,t} = P_t$, and writing in the sequence space yields

$$d\boldsymbol{C_H} = (1-\alpha)d\boldsymbol{C}.$$

Inserting eq. (24) into this yields

$$d\boldsymbol{C_H} = (1-\alpha)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}).$$

A.3 A Fixed Exchange Rate

In this Appendix, I consider replacing the monetary policy rule in eq. (10) with a fixed exchange rate:

$$E_t = E_{ss}.$$

Figure A.1 replicates Figure 3 with a fixed exchange rate.

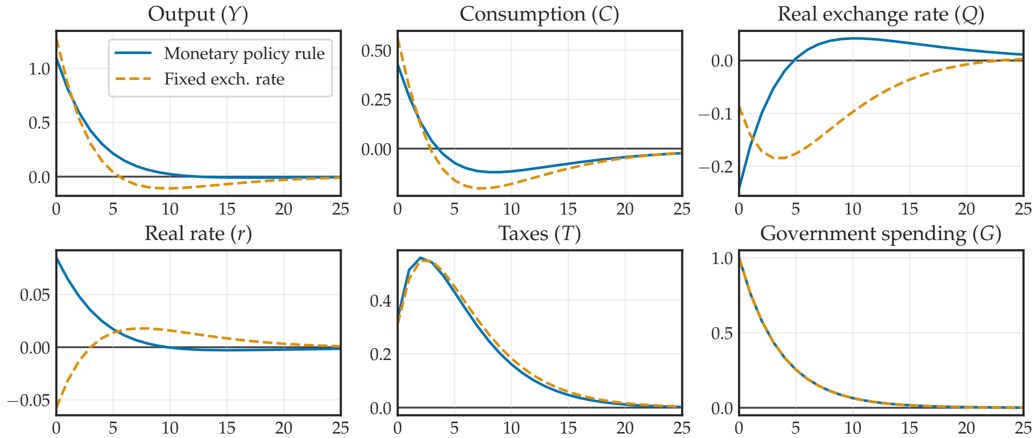

Figure A.1: IRFs to a government spending shock with active monetary policy and a fixed exchange rate

Note: The figure shows $100 \cdot d\boldsymbol{X}$ for various $\boldsymbol{X}$ from the model in Appendix A.1 with both a monetary policy rule as in Figure 3 and a fixed exchange rate. The x-axis shows years after the shock.

A.4 A Representative Agent

The representative agent solves

$$V_t(A_{t-1}) = \max_{C_t, A_t} u(C_t) + \beta \mathbb{E}_t \left[V_{t+1}(A_t) \right], \tag{25}$$

s.t.

$$C_t + A_t = (1 + r_{t-1})A_{t-1} + Z_t, \tag{26}$$

$$\lim_{t \to \infty} q_t A_t = 0. \tag{27}$$

Solving this yields a standard Euler equation, which pins down consumption growth rates for $t = 0, 1, \ldots$:

$$\frac{C_{t+1}}{C_t} = [\beta(1 + r_t)]^{\frac{1}{\sigma}}. \tag{28}$$

Using this with the transversality condition in eq. (27) and budget constraint in eq. (26) gives

$$C_0 = (1 - \beta)\mathcal{I}, \tag{29}$$

where

$$\mathcal{I} = (1 + r_{ss})A_{ss} + \sum_{t=0}^{\infty} q_t Z_t$$

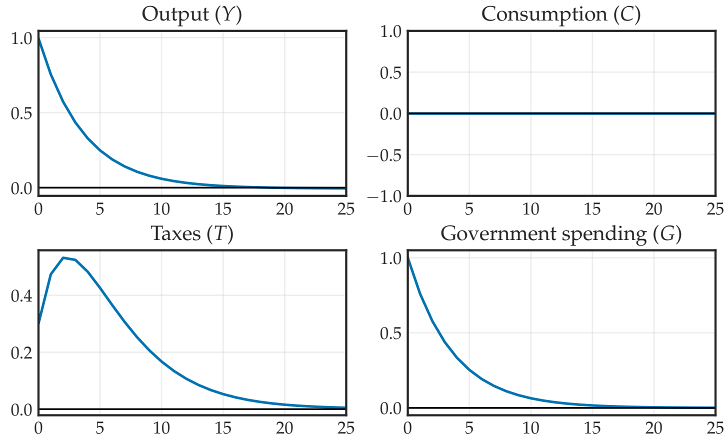

Figure A.2: IRFs to a government spending shock with a representative agent

Note: The figure shows $100 \cdot d\boldsymbol{X}$ for various $\boldsymbol{X}$ from the representative agent model. The x-axis shows years after the shock.

is lifetime income. This pins down the level of consumption.

Since this model satisfies the standard aggregate budget constraint and transversality condition, it follows by Proposition 2 that $\mathcal{M} = 1$. This implies that $\boldsymbol{q}' d\boldsymbol{Z} = 0$, so $dC_0 = 0$ by eq. (29). Since the real interest rate is fixed, it follows from the Euler equation in eq. (28) that $dC_t = 0$ for all t, which finally implies $dY_t = dG_t$. Figure A.2 shows this.

B Proofs

B.1 Proof of Proposition 1

I start by showing $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$, following Auclert et al. (2023). Since I both consider the case of a time-varying interest rate, r_t, and constant interest rate, $r_t = r$, I prove this for a time-varying real interest rate, which nests the case of a fixed real interest rate.

Iterating on the aggregate budget constraint in eq. (5) yields

$$q_t A_t = (1 + r_{ss}) A_{ss} + \sum_{s=0}^{\infty} q_s (Z_s - C_s),$$

where

$$q_t \equiv (1 + r_0)^{-1} (1 + r_1)^{-1} \dots (1 + r_{t-1})^{-1}, \quad \text{for } t = 1, 2, \dots \tag{30}$$
$$q_0 \equiv 1.$$

Using the transversality condition, $\lim_{t\to\infty} q_t A_t = 0$, gives[6]

$$\sum_{t=0}^{\infty} q_t C_t = (1 + r_{ss}) A_{ss} + \sum_{t=0}^{\infty} q_t Z_t. \tag{31}$$

Taking the derivative w.r.t. Z_s yields

$$\sum_{t=0}^{\infty} q_t M_{t,s} = q_s.$$

Evaluating in the steady state then gives

$$\sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^t} = \frac{1}{(1+r)^s}.$$

This in turn implies that

$$\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'.$$

Re-arranging yields

$$\boldsymbol{q}'(\boldsymbol{I} - \boldsymbol{M}) = \boldsymbol{0}.$$

If $(\boldsymbol{I} - \boldsymbol{M})$ had an inverse, multiplying by this from the right would imply that $\boldsymbol{q}' = \boldsymbol{0}$, which is not true, so $(\boldsymbol{I} - \boldsymbol{M})$ cannot have an inverse. This implies that there is no unique solution for $d\boldsymbol{Y}$ in the closed economy, which is a result from Auclert et al. (2023).

Consider now $\boldsymbol{I} - (1-\alpha)\boldsymbol{M}$. To start, consider the following object:

$$[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]\,\boldsymbol{x} = \boldsymbol{0},$$

for $\boldsymbol{x} \in \ell^{\infty}$. If $\boldsymbol{I} - (1-\alpha)\boldsymbol{M}$ is invertible, the only solution is $x = 0$. Otherwise, some other solution exists. Re-write the above to find that

$$(1-\alpha)\boldsymbol{M}\boldsymbol{x} = \boldsymbol{x}.$$

6. This is the transversality with a time-varying real interest rate. It collapses to the one in eq. (4) when $r_t = r$.

Multiply by $\boldsymbol{q}'$ to get that

$$(1-\alpha)\boldsymbol{q}'\boldsymbol{x} = \boldsymbol{q}'\boldsymbol{x}.$$

For $\alpha > 0$, the only solution is $\boldsymbol{x} = \boldsymbol{0}$, so $\boldsymbol{I} - (1-\alpha)\boldsymbol{M}$ is invertible. This implies that there is a unique solution for $d\boldsymbol{Y}$ in the SOE.

B.2 Proof of Proposition 2

Multiply the IIKC in eq. (3) by $\boldsymbol{q}'$:

$$\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha)\boldsymbol{q}'(d\boldsymbol{Y} - d\boldsymbol{T}) + \boldsymbol{q}'d\boldsymbol{G},$$

where I used $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$ from Appendix B.1. Solving for $\boldsymbol{q}'d\boldsymbol{Y}$ yields

$$\boldsymbol{q}'d\boldsymbol{Y} = \frac{\boldsymbol{q}'d\boldsymbol{G} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T}}{\alpha}. \tag{32}$$

Note now that iterating on the government's budget constraint in eq. (16) and using $\lim_{t\to\infty} B_t/(1+r)^t = 0$ implies

$$\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G}. \tag{33}$$

Using this in eq. (32) yields

$$\boldsymbol{q}'d\boldsymbol{Y} = \boldsymbol{q}'d\boldsymbol{G}.$$

Dividing by $\boldsymbol{q}'d\boldsymbol{G}$ and using the definition of the cumulative fiscal multiplier from eq. (6) then finally gives

$$\mathcal{M} = 1.$$

B.3 Proof of Proposition 3

Start with eq. (32):

$$\boldsymbol{q}'d\boldsymbol{Y} = \frac{\boldsymbol{q}'d\boldsymbol{G} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T}}{\alpha}.$$

Insert eq. (8) to get

$$\boldsymbol{q}'d\boldsymbol{Y} = \frac{\boldsymbol{q}'d\boldsymbol{G} - (1-\alpha)f\boldsymbol{q}'d\boldsymbol{G}}{\alpha}.$$

Divide by $\boldsymbol{q}'d\boldsymbol{G}$ to arrive at the cumulative fiscal multiplier:

$$\mathcal{M} = \frac{1}{\alpha} - (1-\alpha)f\frac{1}{\alpha}.$$

Re-write this to arrive at

$$\mathcal{M} = 1 + \frac{(1-\alpha)(1-f)}{\alpha}.$$

B.4 Proof of Proposition 4

Start by pre-multiplying the multi-country IIKC from eq. (9) by $(\boldsymbol{q}', \boldsymbol{q}')$:

$$\begin{pmatrix} \boldsymbol{q}'d\boldsymbol{Y} \\ \boldsymbol{q}'d\boldsymbol{Y^*} \end{pmatrix} = \begin{pmatrix} 1-\alpha-\epsilon & \alpha^* \\ \alpha & 1-\alpha^*-\epsilon^* \end{pmatrix} \begin{pmatrix} \boldsymbol{q}'(d\boldsymbol{Y}-d\boldsymbol{T}) \\ \boldsymbol{q}'d\boldsymbol{Y^*} \end{pmatrix} + \begin{pmatrix} \boldsymbol{q}'d\boldsymbol{G} \\ 0 \end{pmatrix}.$$

The first block for $\boldsymbol{q}'d\boldsymbol{Y}$ is:

$$\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha-\epsilon)\boldsymbol{q}'(d\boldsymbol{Y}-d\boldsymbol{T}) + \alpha^* d\boldsymbol{Y}^* + \boldsymbol{q}'d\boldsymbol{G}.$$

Solving for $\boldsymbol{q}'d\boldsymbol{Y}^*$ in the second blocks yields:

$$\boldsymbol{q}'d\boldsymbol{Y^*} = \frac{\alpha}{\alpha^*+\epsilon^*}\boldsymbol{q}'(d\boldsymbol{Y}-d\boldsymbol{T}),$$

if $\alpha+\epsilon \neq 0$, which holds due to $\alpha > 0$ and $\epsilon \geq 0$. Inserting this into the expression for $\boldsymbol{q}'d\boldsymbol{Y}$ and collecting terms yields

$$\boldsymbol{q}'d\boldsymbol{Y} = \left(1-\alpha-\epsilon+\alpha^*\frac{\alpha}{\alpha^*+\epsilon^*}\right)\boldsymbol{q}'(d\boldsymbol{Y}-d\boldsymbol{T}) + \boldsymbol{q}'d\boldsymbol{G}.$$

Solving for $\boldsymbol{q}'d\boldsymbol{Y}$ yields

$$\boldsymbol{q}'d\boldsymbol{Y} = \frac{\boldsymbol{q}'d\boldsymbol{G} - \left(1-\alpha-\epsilon+\alpha^*\frac{\alpha}{\alpha^*+\epsilon^*}\right)\boldsymbol{q}'d\boldsymbol{T}}{\alpha+\epsilon-\alpha^*\frac{\alpha}{\alpha^*+\epsilon^*}}$$

if

$$\alpha + \epsilon - \alpha^* \frac{\alpha}{\alpha^* + \epsilon^*} \neq 0. \tag{34}$$

Using $\boldsymbol{q}' d\boldsymbol{G} = \boldsymbol{q}' d\boldsymbol{T}$ then finally yields

$$\mathcal{M} = 1.$$

Note that this only holds if condition (34) is satisfied. By re-arranging the condition and using $\alpha > 0$ and α^*, it follows that condition (34) fails only when $\epsilon = \epsilon^* = 0$. Thus, it follows that $\mathcal{M} = 1$ if either $\epsilon > 0$ or $\epsilon^* > 0$.

B.5 Proof of Proposition 5

B.5.1 Domestic Consumption of Domestic Goods

Linearizing CPI in eq. (15) yields

$$dP_t = \alpha dP_{F,t} + (1 - \alpha) dP_{H,t}. \tag{35}$$

Next, linearizing the nominal exchange rate in eq. (18) yields

$$dE_t = dP_{F,t} - dP_t^*. \tag{36}$$

Similarly, linearizing the real exchange rate in eq. (19) implies that

$$dQ_t = dE_t + dP_t^* - dP_t. \tag{37}$$

Inserting eq. (36) into eq. (37) yields

$$dQ_t = dP_{F,t} - dP_t.$$

Solving for $dP_{F,t}$, I find that

$$dP_{F,t} = dQ_t + dP_t.$$

Inserting this into the linearized CPI in eq. (35) and solving for $dP_{H,t} - dP_t$, I find that

$$dP_{H,t} - dP_t = -\frac{\alpha}{1 - \alpha} dQ_t. \tag{38}$$

Using this in eq. (14) linearized yields:

$$dC_{H,t} = (1-\alpha)dC_t + \eta\alpha C_{ss} dQ_t. \tag{39}$$

B.5.2 Foreign Consumption of Domestic Goods

Linearizing PCP gives

$$dP^*_{H,t} = dP_{H,t} - dE_t.$$

Subtracting eq. (37) solved for dP^*_t gives

$$dP^*_{H,t} - dP^*_t = dP_{H,t} - dP_t - dQ_t.$$

Inserting eq. (38) gives

$$dP^*_{H,t} - dP^*_t = -\frac{1}{1-\alpha} dQ_t.$$

Using this in eq. (17) linearized yields:

$$dC^*_{H,t} = \alpha dC^*_t + \frac{\alpha}{1-\alpha}\gamma C^*_{ss} dQ_t. \tag{40}$$

B.5.3 Jacobian Relation

Taking the derivative of eq. (31) w.r.t. r_s yields:

$$\sum_{t=0}^{\infty} q_t M^r_{t,s} + \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s} C_t = \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s} Z_t,$$

where $M^r_{t,s} = \partial C_t / \partial r_s$ is element (t, s) of the Jacobian $\boldsymbol{M^r}$. Evaluating in the steady state gives:

$$\sum_{t=0}^{\infty} \frac{M^r_{t,s}}{(1+r_{ss})^t} + C_{ss} \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss} = Z_{ss} \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss},$$

where "$|_{ss}$" denotes evaluation in the steady state. Note that evaluating the aggregate budget constraint from eq. (5) in the steady state yields

$$C_{ss} - Z_{ss} = r_{ss} A_{ss},$$

such that

$$\sum_{t=0}^{\infty} \frac{M^r_{t,s}}{(1+r)^t} = -r_{ss} A_{ss} \sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss}. \tag{41}$$

By differentiating the definition of q_t in eq. (30), I find that

$$\frac{\partial q_t}{\partial r_s}\Big|_{ss} = -\frac{1}{(1+r_{ss})^{t+1}}, \quad \text{for } s = 0, 1, \ldots, t-1$$
$$\frac{\partial q_t}{\partial r_s}\Big|_{ss} = 0, \quad \text{for } s = t, t+1, \ldots.$$

Evaluating this in the steady state gives

$$\sum_{t=0}^{\infty} \frac{\partial q_t}{\partial r_s}\Big|_{ss} = -\sum_{t=s}^{\infty} \frac{1}{(1+r_{ss})^{t+2}} = -\frac{1}{r_{ss}} \frac{1}{(1+r_{ss})^{s+1}},$$

using the standard formula for the sum of a geometric series in the second equality. Inserting this in eq. (41) gives

$$\sum_{t=0}^{\infty} \frac{M^r_{t,s}}{(1+r)^t} = \frac{A_{ss}}{1+r_{ss}} \frac{1}{(1+r_{ss})^s},$$

which finally implies

$$\boldsymbol{q}'\boldsymbol{M}^{\boldsymbol{r}} = \frac{A_{ss}}{1+r_{ss}} \boldsymbol{q}'. \tag{42}$$

B.5.4 Main Proof

The linearized consumption function is

$$d\boldsymbol{C} = \boldsymbol{M} d\boldsymbol{Z} + \boldsymbol{M}^{\boldsymbol{r}} d\boldsymbol{r} \tag{43}$$

with Jacobians

$$\boldsymbol{M} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{Z}}, \quad \boldsymbol{M}^{\boldsymbol{r}} \equiv \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}}.$$

Insert now $W_t = P_{H,t}$ into the definition of labor income to get that

$$Z_t = \frac{P_{H,t}}{P_t} Y_t - T_t.$$

Taking a first-order approximation of this yields

$$dZ_t = dY_t - dT_t + dP_{H,t} - dP_t = dY_t - \frac{\alpha}{1-\alpha} dQ_t.$$

where I used eq. (38) in the second equality. Inserting this into the linearized consumption function in eq. (43), it follows that

$$d\boldsymbol{C} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) - \frac{\alpha}{1-\alpha}\boldsymbol{M}d\boldsymbol{Q} + \boldsymbol{M}^{\boldsymbol{r}}d\boldsymbol{r}. \tag{44}$$

I use this shortly. To do this, turn next to linearized goods market clearing:

$$d\boldsymbol{Y} = d\boldsymbol{C_H} + d\boldsymbol{C_H^*} + d\boldsymbol{G}.$$

Insert eq. (39) and eq. (40) and gathering terms yields:

$$d\boldsymbol{Y} = (1-\alpha)d\boldsymbol{C} + \left[\eta\alpha C_{ss} + \gamma\frac{\alpha}{1-\alpha}C_{ss}^*\right] d\boldsymbol{Q} + d\boldsymbol{G},$$

using $d\boldsymbol{C}^* = 0$. Inserting eq. (44) yields

$$d\boldsymbol{Y} = (1-\alpha)\left[\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) - \frac{\alpha}{1-\alpha}\boldsymbol{M}d\boldsymbol{Q} + \boldsymbol{M}^{\boldsymbol{r}}d\boldsymbol{r}\right] + \left[\eta\alpha + \gamma\frac{\alpha}{1-\alpha}\right] C_{ss}d\boldsymbol{Q} + d\boldsymbol{G}$$

using $C_{ss} = C_{ss}^*$. Isolating $d\boldsymbol{Y}$ then gives

$$\left[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}\right] d\boldsymbol{Y} = (1-\alpha)\left[\boldsymbol{M}^{\boldsymbol{r}}d\boldsymbol{r} - \boldsymbol{M}d\boldsymbol{T}\right] + \left[\frac{\alpha}{1-\alpha}\chi\boldsymbol{I} - \alpha\boldsymbol{M}\right] d\boldsymbol{Q} + d\boldsymbol{G},$$

where

$$\chi \equiv \left[\eta(1-\alpha) + \gamma\right] C_{ss}.$$

Pre-multiplying by $\boldsymbol{q}'$ gives:

$$\alpha\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha)\frac{A_{ss}}{1+r_{ss}}\boldsymbol{q}'d\boldsymbol{r} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T} + \left[\frac{\alpha}{1-\alpha}\chi - \alpha\right]\boldsymbol{q}'d\boldsymbol{Q} + \boldsymbol{q}'d\boldsymbol{G}, \tag{45}$$

where I used eq. (42). Note next that $\boldsymbol{q}'d\boldsymbol{G} \neq \boldsymbol{q}'d\boldsymbol{T}$ when r_t is time-varying. In this case, iterating on the government's budget constraint and using $\lim_{t\to\infty} B_t/(1+r)^t$

yields

$$\sum_{t=0}^{\infty} q_t \text{PD}_t + (1+r)B_{ss} = 0.$$

Taking a first order approximation and using $\text{PD}_t = (P_{H,t}/P_t)G_t - T_t$ gives

$$\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G} + G_{ss}\boldsymbol{q}'(d\boldsymbol{P_H} - d\boldsymbol{P}) + \text{PD}_{ss}\sum_{t=0}^{\infty} dq_t,$$

where dq_t is a perturbation of q_t around $(1+r)^{-t}$. Note that the case of constant real interest rate gives $dq_t = 0$ and $d\boldsymbol{P_H} = d\boldsymbol{P} = 0$, so thus $\boldsymbol{q}'d\boldsymbol{G} = \boldsymbol{q}'d\boldsymbol{T}$. Using eq. (38) gives

$$\boldsymbol{q}'d\boldsymbol{T} = \boldsymbol{q}'d\boldsymbol{G} - G_{ss}\frac{\alpha}{1-\alpha}\boldsymbol{q}'d\boldsymbol{Q} + \text{PD}_{ss}\sum_{t=0}^{\infty} dq_t \tag{46}$$

Inserting eq. (46) into eq. (45) yields

$$\alpha\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha)\frac{A_{ss}}{1+r_{ss}}\boldsymbol{q}'d\boldsymbol{r} + \left[\frac{\alpha}{1-\alpha}\chi - \alpha\right]\boldsymbol{q}'d\boldsymbol{Q} + \alpha\boldsymbol{q}'d\boldsymbol{G}$$
$$+ G_{ss}\alpha\boldsymbol{q}'d\boldsymbol{Q} - (1-\alpha)\text{PD}_{ss}\sum_{t=0}^{\infty} dq_t,$$

where I used eq. (42). Dividing by $\alpha\boldsymbol{q}'d\boldsymbol{G}$ finally gives:

$$\mathcal{M} = 1 + \frac{1-\alpha}{\alpha}\frac{A_{ss}}{1+r}\frac{\boldsymbol{q}'d\boldsymbol{r}}{\boldsymbol{q}'d\boldsymbol{G}} + \left[\frac{\chi}{1-\alpha} - 1\right]\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}} + G_{ss}\frac{\boldsymbol{q}'d\boldsymbol{Q}}{\boldsymbol{q}'d\boldsymbol{G}} - \frac{1-\alpha}{\alpha}\text{PD}_{ss}\sum_{t=0}^{\infty} dq_t.$$

B.6 Proof of Proposition 6

Taking a first-order approximation of the iterated government's budget constraint in eq. (11) and taking the limit of the as $t \to \infty$ yields

$$\lim_{t\to\infty}\frac{dB_t}{(1+r)^t} = \boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) - \tau\boldsymbol{q}'d\boldsymbol{Y}, \tag{47}$$

Inserting the definition of ν yields

$$\lim_{t\to\infty}\frac{dB_t}{(1+r)^t} = (1-\nu)\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}).$$

This shows directly that $\nu = 1$ implies $\lim_{t\to\infty} B_t/(1+r)^t = 0$. Note further that the right-hand side of eq. (47) is $\boldsymbol{q}'d\boldsymbol{PD}$. Thus, $\nu = 1$ also implies $\boldsymbol{q}'d\boldsymbol{PD} = 0$.

B.7 Proof of Proposition 7

Iterating on the government's budget constraint in eq. (16) yields

$$B_t = (1+r)^{t+1}B_{ss} + \sum_{s=0}^{t}(1+r)^{t-s}\mathrm{PD}_s.$$

Dividing by $(1+r)^t$ gives

$$\frac{B_t}{(1+r)^t} = (1+r)B_{ss} + \sum_{s=0}^{t}\frac{\mathrm{PD}_s}{(1+r)^s}.$$

Taking a first-order approximation finally yields

$$\frac{dB_t}{(1+r)^t} = \sum_{s=0}^{t}\frac{d\mathrm{PD}_s}{(1+r)^s}.$$

The Keynesian cross is now

$$d\boldsymbol{Y} = (1-\alpha)(1-\tau)\boldsymbol{M}d\boldsymbol{Y} - (1-\alpha)Md\boldsymbol{T} + d\boldsymbol{G}.$$

Multiply by $\boldsymbol{q}'$:

$$\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha)(1-\tau)\boldsymbol{q}'d\boldsymbol{Y} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T} + \boldsymbol{q}'d\boldsymbol{G}.$$

Thus,

$$(\tau + \alpha - \alpha\tau)\boldsymbol{q}'d\boldsymbol{Y} = \boldsymbol{q}'d\boldsymbol{G} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T}.$$

Consider first $d\boldsymbol{G} = 0$ and $\tau > 0$. The Keynesian cross gives

$$\boldsymbol{q}'d\boldsymbol{Y} = \frac{1-\alpha}{\alpha\tau - \tau - \alpha}\boldsymbol{q}'d\boldsymbol{T}.$$

Insert this into ν to get

$$\nu_T = \frac{(1-\alpha)\tau}{(1-\alpha)\tau + \alpha}.$$

When $\alpha = 0$, it follows that

$$\nu_T = 1.$$

When instead $d\boldsymbol{T} = 0$ and still $\tau > 0$, the Keynesian cross gives

$$\boldsymbol{q}' d\boldsymbol{Y} = \frac{1}{(1-\alpha)\tau + \alpha} \boldsymbol{q}' d\boldsymbol{G}.$$

Insert this into ν to get

$$\nu_G = \frac{\tau}{(1-\alpha)\tau + \alpha}.$$

While differing by a factor $1 - \alpha$, it shares all the properties of ν for lump sum taxes: $\nu = 1$ when $\alpha = 0$, ν is decreasing in α, and $\nu \in (0, 1)$ when $\alpha \in (0, 1)$ and $\tau \in (0, 1)$.

For the closed economy, $\nu = 1$ can be observed almost immediately using an alternative argument. Start by observing that

$$\lim_{t \to \infty} \frac{B_t}{(1+r)^t} = \boldsymbol{q}' d\mathbf{PD} = (1 - \nu) \boldsymbol{q}' d\boldsymbol{G},$$

for $d\boldsymbol{T} = 0$. By asset market clearing, $B_t = A_t$, and I know that $\lim_{t \to \infty} \frac{A_t}{(1+r)^t} = 0$. This then implies that $(1 - \nu) \boldsymbol{q}' d\boldsymbol{G} = 0$, which can only hold when $\nu = 1$ (because $\boldsymbol{q}' d\boldsymbol{G} > 0$). The equivalent argument holds for $d\boldsymbol{G} = 0$ and $d\boldsymbol{T} > 0$.